ECO 5350 T. Fomby
Intro. Econometrics Spring 2021

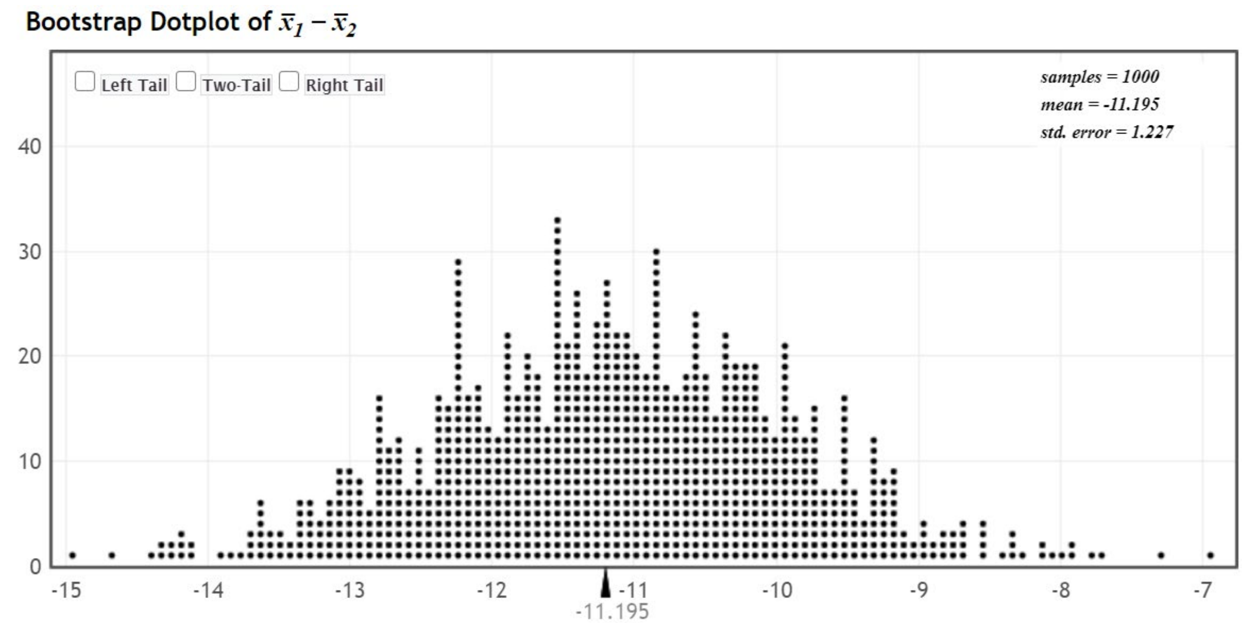
EXERCISE 4
KEY

Purpose: **To re-examine the data of Exercise 3 using Bootstrap and Randomization tests of hypotheses. These tests of the Exercise 3 data can be conducted by means of the Google Chrome STATKEY Applet program found at www.lock5stat.com. This exercise is due on Thursday, February 18 at 5:00 pm CST on Canvas.**

Go to the Assignment section of Exercise 4 on Canvas and download the PPTX file **"September 1 outline.pptx"** for a specific discussion of the Bootstrap and Randomization tests and the pdf file **"Intro to STATKEY.pdf"** for more specifics on the STATKEY website. Read both of these files in preparation for working on this "difference-in-means" statistical problem using the STATKEY Applet and the data of Exercise 3. I will also be discussing the STATKEY website and its tests during class lecture. The data that you are to upload into the STATKEY program is contained in **Exercise 4 Data.csv**. It can be found in the Exercise 4 Assignment in Canvas. The first column is X, that is the data, and the second column is I which is the indicator (categorial) function for the C (control) versus T (treatment) groups.

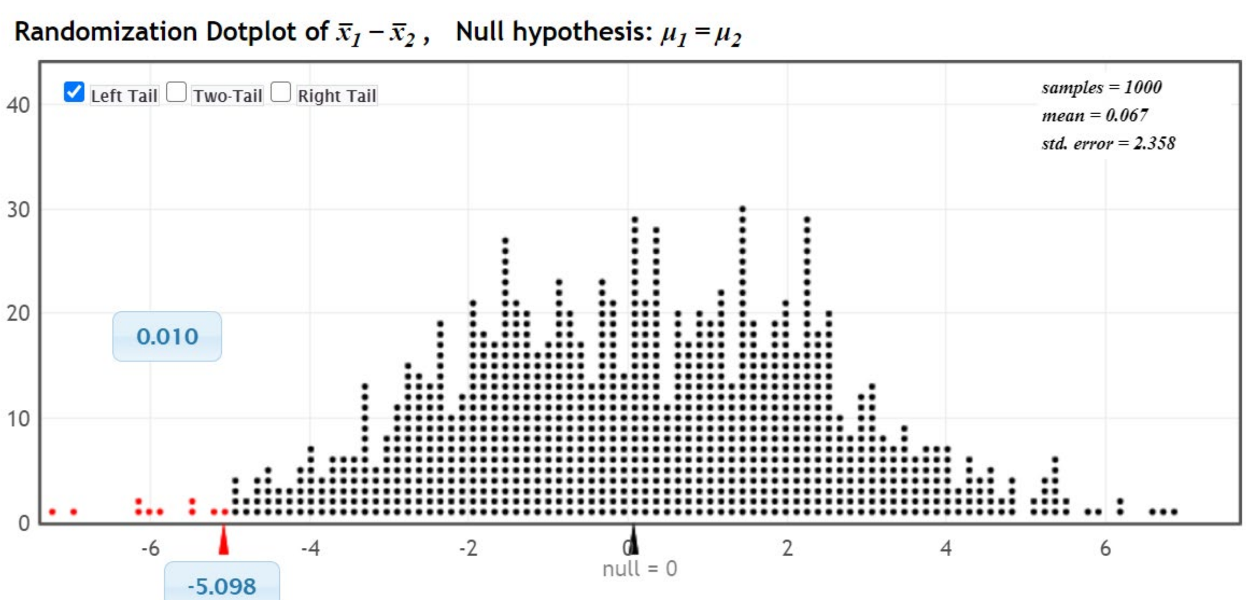
NOTE: In the parts (a) and (b) below, the distributions you generate may be slightly different from what I get or what your classmates get because a random number generator is used to generate the samples and its starting value is randomly selected. However, the conclusions you draw will probably be the same that I draw because the sample size of 1,000 is quite large.

(a) Use STATKEY to create a Bootstrap Distribution using B = 1,000 Bootstrap samples. Like in Exercise 3, remember the null hypothesis is $H_0: \mu_1 - \mu_2 = 0$ versus $H_1: \mu_1 - \mu_2 < 0$. Take a snippet of the Bootstrap distribution that you have generated and turn it in with this exercise. Is the zero value (null value) anywhere within the Bootstrap distribution? What is your conclusion?



Answer: You can see that the Bootstrap distribution does not contain 0 or anything near it. Therefore, we conclude that the null hypothesis is false and we accept the alternative hypothesis that the Treatment mean is greater than the Control mean.

(b) Use STATKEY to create a **Randomization test distribution** for the data of Exercise 3 using **1,000 randomization runs**. Again, recall the null hypothesis is $H_0: \mu_1 - \mu_2 = 0$ versus $H_1: \mu_1 - \mu_2 < 0$. In the Randomization distribution **set the test to be a left-tail test and set the p-value to be 0.01**. Is the original difference in the two samples in the acceptance region or rejection region of the Randomization test distribution? Take a snippet of the Randomization distribution and turn it in with this exercise. What is your conclusion?



Answer: As we can see from the STATKEY results (not shown here) is equal to -11.2 = $\bar{X}_c - \bar{X}_T$ = 9.099 – 20.299. This number is obviously in the 1% rejection region because it is to the left of the 1% critical value of -5.098. (Remember your critical value may be slightly different from mine because of the random number difference.) We, therefore, reject the null hypothesis of equal means and accept the alternative hypothesis that the treatment group mean is greater than the control group mean.

(c) How do the results of the Bootstrap and Randomization tests compare with the results you obtained from the parametric t-test that you conducted in Exercise 3? Explain your answer.

Answer: If you will recall, the parametric t-test of Exercise 3 produced a very negative t-value whose p-value was less than 0.01. Therefore, we rejected the null hypothesis of equal means and accepted the alternative hypothesis that the treatment group mean is greater than the control group mean. We got the same results using the Bootstrap and Randomization tests. (I might note that this coincidence of same results does not always happen and, therefore, one has to be less confident in declaring a unanimous conclusion when this occurs.)